1. The "Rayleigh Criterion", for the ability to resolve two images in presence of diffraction, is that the diffraction maximum of one image is at the first diffraction minimum of the other image. For a circular aperture this comes out to require an angular separation given by

$$\Theta_m = 1.22 \frac{\lambda}{D}$$

where $D$ is the aperture diameter.

a) for $\lambda = 550$ nm and $D = 0.5 \times 10^{-2}$ m

$$\Theta_m = 1.22 \times 550 \times 10^{-9} / 0.5 \times 10^{-2} m$$

$$= 1.032 \times 10^{-5} \text{ rad}$$

at a distance of 1 km, this corresponds to a separation of 1.032 cm

b) for same $\lambda$ and $D = 2.5$ mm

$$\Theta_m = 2.68 \times 10^{-4} \text{ rad}$$

$\Rightarrow$ separation $= 26.8$ cm.
2. Course pack #11

a) $T_{\pi} = 2.6 \times 10^{-8}$ s. This is like a clock ticking on the moving particle. "moving clocks tick slow," so in the lab frame, the "clock" on the particle appears slow by a factor of $\gamma$

$$T_{lab} = \gamma T_{\pi} = \frac{1}{\sqrt{1-0.95^2}} T_{\pi}$$

$$= 3.2 \times 2.6 \times 10^{-8} s = 8.32 \times 10^{-8} s$$

b) distance travelled in the lab

$$d = T_{lab} \cdot V_{lab} = 8.32 \times 10^{-8} s \cdot 0.95 \cdot 3 \times 10^8 m/s$$

$$= 23.7 m$$

c) the $\pi$ meson sees the whole laboratory going the other way at $V = -0.95c$.

d) in the $\pi$ meson frame, the $\pi$ is at rest, the lab is zipping by. In the lifetime of the meson in its rest frame (the "proper time"), the amount of lab that goes by is

$$l = 2.6 \times 10^{-8} s \cdot 0.95 c$$

$$= 7.41 m$$

but since a "moving yardstick appears shorter", we know that this is the Lorentz contracted length of the lab

$$l = \frac{1}{\gamma} l'$$

the true length of the lab is therefore

$$l' = \gamma l = 3.2 \times 7.41 = 23.7 m$$

so it has to be to agree with part 6.
3. at rest

$l'_{\text{lincoln}} = l'_{b}$

$l'_{\text{beetle}} = l'_{b'} + \frac{1}{2} l'_{l}$

(a) in motion, each length is shortened by the $\gamma$ factor related to their velocity

\[
l_b = \frac{1}{\gamma_b} l'_b
\]

\[
l_l = \frac{1}{\gamma_l} l'_l
\]

in the car's frame $l_b = l_l$, thus

\[
\frac{1}{\gamma_b} l'_b = \frac{1}{\gamma_l} l'_l
\]

\[
\gamma_l = \gamma_b \left( \frac{l'_b}{l'_b} \right) = 2 \gamma_b
\]

but we know $\gamma_b$ since the beetle is clicked at $v = 0.99c$

\[
\gamma_b = \frac{1}{\sqrt{1 - (0.99)^2}} \approx 1.15
\]

\[
\gamma_l = 2 \times 1.15 = 2.3
\]

solving

\[
\gamma_l = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \Rightarrow v = c \sqrt{1 - \frac{1}{\gamma^2}} = c \sqrt{1 - \frac{1}{2.3^2}} = 0.99c
\]

(b) [Diagram]

one way to look at this: the bullet has velocity $u'$ in the Harley frame, transform to the rest frame to get
u, which must exceed velocity of the Lincoln in that frame \((N_L=0.9c)\) in order to hit it. The minimum bullet velocity is then

\[
U = \frac{U' + N_L}{1 + \frac{U' N_L}{c^2}}
\]

Solve for \(U'\):

\[
U' + N_L = N_L \left[1 + \frac{U' N_L}{c^2}\right]
\]

\[
U' \left[1 - \frac{V_L N_L}{c^2}\right] = N_L - N_H
\]

\[
U' = \frac{N_L - N_H}{1 - \frac{V_L N_L}{c^2}} = \frac{(0.9 - 0.8)c}{1 - (0.9)(0.8)}
\]

\[
U' = 0.36c
\]

Minimum muzzle velocity is 0.36c.

Alternatively, we can transform to the rest frame of the Lincoln, see what velocity of Lincoln is in that frame, and require the bullet to be faster. The relativistic subtraction of velocities gives:

\[
V_L' = \frac{N_L - N_H'}{1 + \frac{N_L N_H'}{c^2}} \text{ same as here!}
\]

\[
= 0.36c
\]

In the Harley frame, the Lincoln is going 0.36c, and the bullet velocity must exceed this.
In the "earth" frame the clairvoyant and his sister have coordinates \((x_c, t_c)\) and \((x_s, t_s)\). But the sister at the origin, and the clairvoyant at \(x_c = l\); if they experience something simultaneously, the two events in that frame are at spacetime coordinates

\[
(x_s, t_s) = (0, t)
\]

\[
(x_c, t_c) = (l, t)
\]

\[S - S' - N\]

\[0, t\]

\[l, t\]

The helicopter moving with \(v\) to the right, defines a frame \(S'\). We can use the Lorentz transformation to transform the two events to that frame.

\[t_s' = Y(t_s - \frac{v}{c^2} x_s) = Y + s = s +
\]

\[t_c' = Y(t_c - \frac{v}{c^2} x_c) = Y(t - \frac{v}{c^2} l) = Y - \frac{v}{c^2} l
\]

In the helicopter frame, the clairvoyant event seems to happen first, followed by the sister event, with time difference

\[\Delta t = t_s' - t_c' = \frac{v}{c^2} l\]

For \(v = \frac{12}{13} c\), \(Y = \frac{13}{5}\), and \(\Delta t = 4 \times 10^{-5}\) sec = 4 msec.

In this frame, the clairvoyant acoustically reacts before the sister acts!
5. 
(a) 
\[ ^{222}_{86}\text{Ra} \rightarrow ^{222}_{86}\text{Rn} + ^{4}_{2}\text{He} \]

Masses

\[ 226.0254 \text{ amu} \quad 222.0175 \text{ amu} \quad 4.0026 \text{ amu} \]

\[ \Delta m = 226.0254 - 222.0175 - 4.0026 \]

\[ = 0.0053 \text{ amu} \times \frac{1.66 \times 10^{-27} \text{ kg}}{\text{amu}} \]

\[ = 8.8 \times 10^{-30} \text{ kg} \]

\[ \Delta E = \Delta mc^2 = 8.8 \times 10^{-30} \times (3 \times 10^8)^2 = 7.91 \times 10^{-13} \text{ J} \]

\[ \frac{1 \text{ kg Ra}}{226 \text{ g/mole}} = 4.42 \text{ moles} \]

1 kg of Ra decays is then

\[ 4.42 \times 6.023 \times 10^{23} \times 7.91 \times 10^{-13} \text{ J} \]

\[ = 2.1 \times 10^{12} \text{ J}! \]

(b) 
\[ P_{\text{max}} = 4 \times 10^{26} \text{ W} = 4 \times 10^{26} \text{ J/sec} \]

\[ m = \frac{E}{c^2} \]

\[ \Delta m = \frac{1}{c^2} \Delta E = \frac{4 \times 10^{26}}{(3 \times 10^8)^2} = 4.4 \times 10^{-9} \text{ kg/sec} \]
Say the source is moving away, then the distance between
the wave fronts gets stretched out by the distance the source
moves in one period. In the observer's frame, the Doppler
shifted wavelength is

\[ \lambda_d = \lambda + \nu - T \]
\[ = CT + \nu - T \]
\[ = (C+\nu)T \]

Now, the time in the observer's frame is dilated with respect
to time in the rest frame of the source. If the period in
the source rest frame is \( T' \), then

\[ T = \gamma T' \]

Thus

\[ \lambda_d = (C+\nu)\gamma T' \]
\[ \frac{c}{f} = \frac{(C+\nu)\gamma}{f'} \]
\[ f = f' \cdot \frac{c}{(C+\nu)\gamma} \]
\[ = f' \cdot \frac{c}{C+\nu} \cdot \frac{\sqrt{1 - \frac{\nu^2}{c^2}}}{1 + \frac{\nu}{c}} = f' \cdot \frac{\sqrt{c^2 - \nu^2}}{c+\nu} \cdot \frac{\sqrt{(C+\nu)(C-\nu)}}{C+\nu} \]
\[ f = f' \cdot \sqrt{\frac{c-\nu}{c+\nu}} \text{ g.e.d.} \]

If the source is moving toward the observer, \( \lambda_d = \lambda - \nu - T \), and
we end up with

\[ f = f' \cdot \sqrt{\frac{c+\nu}{c-\nu}} \text{ also g.e.d.} \]
6b coursework #36

a) just use the Doppler shifted wavelength in observer frame at top of previous page:

\[ \lambda_d = \lambda + \nu/T \]

\[ = \lambda + \frac{\nu}{f} \]

\[ \Delta \lambda = \lambda_d - \lambda = \frac{\nu}{f} \]

\[ \frac{\Delta \lambda}{\lambda} = \frac{\nu}{f \lambda} = \frac{\nu}{c} \quad q.e.d. \]

b) \[ \frac{\Delta \lambda}{\lambda} = \frac{20 \text{nm}}{397 \text{nm}} = 5 \times 10^{-2} \]

\[ \therefore \text{recessional velocity} = 0.05c \]