2.1. Characteristics of the 2 wire transmission line

a. To calculate \( C \) and \( L \) per unit length, see Hinow Chap. 9, Ex. 4 p. 151

\[ V = \sqrt{\frac{C}{L}} = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} \]

Analysis of units:

\[ \left[ \mu_0 \varepsilon_0 \right] \cdot \frac{H \cdot F}{m^2} = \left[ \frac{\text{N} \cdot \text{m}}{A^2 \cdot \text{m}} \right] \cdot \left[ \frac{\text{m}^2}{\text{F} \cdot \text{m}} \right] = \frac{1}{\text{m}^2} \]

\[ \Rightarrow \quad V = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \approx 2.997 \times 10^8 \text{ m/sec} \]

b. \[ \Rightarrow \quad V = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \approx 2.997 \times 10^8 \text{ m/sec} \]

c. \[ Z = \sqrt{\frac{L}{C}} = \frac{1}{\pi} \sqrt{\frac{\mu_0}{\varepsilon_0}} \mu_0 \left( \frac{1}{\lambda} \right) \Omega \]

\[ \left[ \mu_0 \right] \cdot \frac{\text{N}}{\text{A}^2} = \frac{\text{N} \cdot \text{m}}{\text{A}^2 \cdot \text{C}^2} \]

\[ \Rightarrow \quad \frac{V}{A^2} = \text{ohm}^2 \]

d. \[ a = 0.0404 \text{ m} \quad \mu_0 \left( \frac{1}{\lambda} \right) \approx 2.51 \]

\[ b = 0.5 \text{ m} \quad \mu_0 \left( \frac{1}{\lambda} \right) \approx 2.51 \]

\[ C = \frac{\pi \varepsilon_0}{\ln \left( \frac{b}{a} \right)} = \frac{3.14}{2.51} \varepsilon_0 = 1.25 \cdot 8.85 \times 10^{-12} \text{ F/m} = 1.11 \times 10^{-11} \text{ F/m} \]

\[ L = \frac{\mu_0 b^2}{\pi} \mu_0 = \frac{2.51}{3.14} \mu_0 = 3.19 \pi \times 10^{-7} \text{ H} \]

\[ V = C \]

\[ Z = \frac{2.51 \cdot \sqrt{\mu_0}}{3.14} = \frac{2.51 \cdot 377.52}{3.14} = 301.52 \]

\[ \Rightarrow \text{ this simple system modifies vacuum transmission properties by factor of order unity,} \]
\[ B = 10 \text{ cm}^2 \]

\[ T = \frac{10}{2\pi} = \frac{1}{30} \text{ s}^{-1} \]

Generally:

\[ \mathbf{E} \cdot d\mathbf{l} = -N \frac{2}{\pi} \int \mathbf{B} \cdot d\mathbf{s} \]

In the easy geometry of this problem, \( \mathbf{E} \) is tangential to the circle at radius \( r \), and \( \mathbf{B} \) is \( \perp \) to the surface enclosed, so the integrals are trivial.

\[ \mathbf{E} \cdot 2\pi r = -N \frac{2}{\pi} \left[ B \int ds \right] = -\frac{\partial B}{\partial t} \cdot \pi r^2 \]

We don't even care about \( \mathbf{E} \):

\[ \mathbf{E} \cdot \mathbf{B} = -N \pi r^2 \frac{2}{\pi} \left[ B_{0} \sin(wt) \right] \]

\[ = -N \pi r^2 \omega B_{0} \cos(wt) \]

\[ = -10(14 \times 10^{-4}) \cdot 60\pi \cdot 0.5 \cos(60\pi t) \quad \text{m}^2 \text{T}^{-1} \text{V}^{-1} \]

\[ = 1.32 \text{ cos}(60\pi t) \text{ V} \]

**Units:**

\[ \text{eV} = \text{energy} \Rightarrow [V] = \frac{\text{Joules}}{\text{C}} = \frac{N - m}{C} \]

\[ F = qV \Rightarrow [\mathbf{B}] = \text{Tesla} = \frac{N - S}{m^2 \text{C}} \]

Here:

\[ m^2 \text{T}^{-1} \text{V}^{-1} = \text{m}^2 \text{s}^{-1} \cdot \frac{N - S}{m^2 \text{C}} \]

\[ \frac{N - S}{m^2 \text{C}} = \frac{N - m}{C} \cdot \text{V} \]

\[ 1.32V \]

\[ \frac{1}{130} \text{s} \]
4.3. \( E(z,t) = E_0 \cos(kx-\omega t) \)  
\[ E_0 = 22 \text{ V/m} \quad \lambda = 50 \text{ m} \]

a. \( f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{50 \text{ m}} = 6 \times 10^6 \text{ Hz} \)  
\[ w = 2\pi f = 12\pi \text{ rad/s} \]

This is "short-wave", above AM, below FM

b. as above, but we need to find \( k \)

\[ k = \frac{2\pi}{\lambda} = 0.4\pi \text{ m}^{-1} \]

\[ E(z,t) = \frac{22}{\mu} \cos (.04\pi x - 12\pi t) \]

c. \( B_z = \frac{E_0}{c} = 7.3 \times 10^{-8} \text{ T} \)

\[ \mu_0 B(z,t) = -7.3 \times 10^{-8} \text{ T} \cos (.04\pi x - 12\pi t) \]

d. Note \( B\) points in \(-x\) direction when \( E\) is in \( y\) direction, so that \( E \times B \) is \( E \).

45. a. \( f = \frac{c}{\lambda} \implies f = 4.74 \times 10^{14} \text{ Hz} \), red/orange

b. you are given power and area, so you can calculate intensity which is average of \( S \)

\[ I = \frac{P}{A} = \langle S \rangle = \frac{1}{2} c E^2 \]

\[ \Rightarrow E = \left( \frac{2P}{A \sigma c} \right)^{\frac{1}{2}} = \left[ \frac{2}{\pi \cdot 10^{-6} \cdot 8.85 \times 10^{-12} \cdot 3 \times 10^8} \right]^{\frac{1}{2}} \approx 1900 \text{ V/m} \]

c. Energy in tube of length \( l \) travels through surface \( A \) in time \( \Delta t = \frac{l}{c} \).

\[ E = \langle S \rangle \cdot A \cdot \Delta t \]

\[ = P \Delta t \quad \text{(makes sense!)} \]

\[ = 15 \times 10^{-2} \cdot \frac{1 \text{ m}}{3 \times 10^8 \text{ m/s}} \]

\[ = 50 \text{ pJ} \]
\[ F = |eE| + e\sigma \times B = |eE| + e\frac{\sigma \times B}{c} = |eE| + \frac{\sigma \times E}{c} \]

The magnitude of magnetic term is \( \frac{1}{c} \) smaller than electric term, so magnetic force negligible in non-relativistic limit.

(How do we know it's non-rel? relativistic effects enter when \( E_{\text{kinetic}} = m + \text{mass} = 511 \text{ keV} \) for electron. Since this e.m. wave has amplitude \( 1.9 \text{ kV/m} \), electron energy will be high ~2 keV, so very non-rel.)

With electric force only then; simple minded solution of equation of motion:

\[ F = ma \]

\[ eE \cos (kx - \omega t) = m \ddot{x} \]

(say \( E = E \hat{e}_z \))

Put electron @ \( x = 0 \):

\[ eE \cos (\omega t) = m \ddot{x} \]

\[ \ddot{x} = \frac{eE}{m} \cos (\omega t) \]

\[ \dot{x} = -\frac{eE}{m} \sin (\omega t) + C \Rightarrow V_{\text{max}} = \frac{eE}{mw} \]

\[ x = -\frac{eE}{mw^2} \cos (\omega t) \Rightarrow x_{\text{max}} = \frac{V_{\text{max}}}{\omega} \]

\[ V_{\text{max}} = \frac{1.6 \times 10^{-19}}{9.11 \times 10^{-31} \cdot 2\pi \cdot 4.74 \times 10^{14}} = 0.112 \text{ m/s} \]

\[ x_{\text{max}} = \frac{V_{\text{max}}}{\omega} = 3.76 \times 10^{-17} \text{ m} \] not much!
6. The tangential component of the total electric field at a metallic surface is 0. (Otherwise, charge would move in the conductor to cancel the field.)

b. We are given

\[ \vec{E}_i(x,t) = E_0 \sin(kz - \omega t) \hat{\imath} \]

This is moving in the positive \( z \) direction. The reflected wave moves in the negative \( z \) direction, and must add to above so that \( E_x = 0 \) at the surface. Here, we need to be careful with signs: consider the function \( \sin(kt) \), and let the surface be at \( z = 0 \):

The incident wave is travelling in the positive \( z \) direction toward the conducting surface, so the conductor is in the region \( z > 0 \), and the incident wave is modelled by the piece of this picture at \( z < 0 \). Meanwhile, it should be clear that the waveform "behind" the conductor \( (z > 0) \), if moving in the \(-z\) direction, is just what is needed for the reflected wave, so that \( E_i + E_{\text{reflected}} \) will always equal zero at the surface. So, since we started with \( \sin(kt) \), the reflected wave, moving in the \(-z\) direction is

\[ \vec{E}_r(z,t) = E_0 \sin(kz + \omega t) \hat{\imath} \]
Then, $E_{tot}(z,t) = E_i(z,t) + E_r(z,t)$

\[= E_0 \left[ \sin(kz-wt) + \sin(kz+wt) \right] \hat{i} \]

\[= 2E_0 \sin(kz) \cos(wt) \hat{i} \]

which is a standing wave, with a node at $z=0$, the surface.

For each wave, $\mathbf{E}$ is in phase with $\mathbf{B}$, and has direction such that $\mathbf{E} \times \mathbf{B}$ is the direction of propagation.

\[\mathbf{E_i}(z,t) = E_0 \sin(kz-wt) \hat{j} \]

\[\mathbf{E_r}(z,t) = -E_0 \sin(kz+wt) \hat{j} \]

\[\mathbf{E_{tot}}(z,t) = \mathbf{E_i}(z,t) + \mathbf{E_r}(z,t) \]

\[= E_0 \left[ \sin(kz-wt) - \sin(kz+wt) \right] \hat{j} \]

\[= -2E_0 \cos(kz) \sin(wt) \hat{j} \]

which is a standing wave with an antinode at $z=0$.

B and $E$ are out of phase.
look edge-on at the surface as the incident wave arrives:

the incident electric field parallel to the surface excites a current
that can flow near the surface.

since the skin depth is very small, can think of the current
as flowing in an infinitely thin sheet on the surface, in the \( \pm x \)
direction. define a surface current density \( j \) = current per length in \( y \)
along the surface.

this current is related to the magnetic field at the surface by

Amperes Law:

\[ \oint B \cdot dl = \mu_0 j \]

the “Amperian loop” is shown as the rectangular dotted path in
the drawing. since the normal component of \( B \), as well as
interior value of \( B \) is zero, \( \oint B \cdot dl \) is just \( B \cdot l \), meanwhile, the
current enclosed by this loop is \( j \) times length of loop in
\( y \) direction:

\[ \oint B \cdot dl = B \cdot l - B_x dz + B_{yx} \frac{d}{dx} l + B_{x0} \frac{d}{dx} = \mu_0 j l \]

\[ |\mathbf{B}| = \mu_0 J \]

\[ j = \frac{1}{\mu_0} |\mathbf{B}| \]

\[ j = \frac{-2B_0 \cos(kz) \sin(\omega t)}{\mu_0} \]

\[ j \bigg|_{z=0} = \frac{-2B_0 \sin(\omega t)}{\mu_0} \]

there is an alternating current sheet on the surface, considered
as accelerating charged, this is source of radiation which is
the reflected wave!
the boundary condition that \( E_{\|} = 0 \) at the two surfaces reminds us of standing waves on a string with ends fixed. Our standing wave solution was automatically satisfied at \( z = 0 \):

\[
E(z,+) = 2E_0 \sin(kz) \cos(\omega t)
\]

and can also be fixed to 0 at \( z = L \) if

\[
k = \frac{n\pi}{L}, \quad n = 1, 2, 3, \ldots
\]

then

\[
k = \frac{2\pi f}{\lambda} = \frac{n\pi f}{\lambda} \Rightarrow \lambda = \frac{2L}{n} = \frac{2\text{inches}}{n}, \quad \frac{5.08\text{cm}}{n}
\]

\[
f = \frac{c}{\lambda} = \frac{nc}{2L} = \frac{n \cdot 3 \times 10^8 \text{m/s}}{2 \cdot 2.54\text{cm}} = \frac{n \cdot 5.9 \text{GHz}}{n}
\]

the lowest mode is 5.9 GHz with \( \lambda = 5.08 \text{cm} \).

the 2nd mode, at \( f = 11.8 \text{GHz} \) is what rf types call x-band.
\[ \mathbf{E} = E_0 \cos(\omega t) \mathbf{i} - E_0 \cos(\omega t) \mathbf{j} \quad (at \ z=0) \]

In right handed coordinate system, looking down z-axis:

\[ \frac{E_x}{E_y} = 1, \quad \text{linearly polarized} \]

polarization angle \(-\frac{\pi}{4}\)

or \[ \mathbf{E} = (E_0 \mathbf{i} - E_0 \mathbf{j}) \cos(kx - \omega t) \]

b. \[ \mathbf{E} = E_0 \cos(kz - \omega t) \mathbf{i} + E_0 \cos(kz - \omega t + \frac{\pi}{4}) \mathbf{j} \]

\[ = E_0 \cos(kz - \omega t) \mathbf{i} - E_0 \sin(kz - \omega t) \mathbf{j} \]

\[ E_x^2 + E_y^2 = E_0^2 \Rightarrow E_x, E_y \text{ lie on a circle} \]

Look at \( z = 0 \)

\[ \mathbf{E} = E_0 \cos(-\omega t) \mathbf{i} - E_0 \sin(-\omega t) \mathbf{j} \]

\[ = E_0 \cos(\omega t) \mathbf{i} + E_0 \sin(\omega t) \mathbf{j} \]

This is a right circularly polarized wave

c. \[ \mathbf{E} = E_0 \cos(kz - \omega t) \mathbf{i} + \cos(kz - \omega t + \frac{\pi}{4}) \mathbf{j} \]

Look at \( z = 0 \)

\[ \mathbf{E} = E_0 \left[ \cos(-\omega t) \mathbf{i} + \cos(-\omega t + \frac{\pi}{4}) \mathbf{j} \right] \]

elliptical polarization

Since \( \tan \theta = \frac{1}{E_{x0}^2 - E_{y0}^2} \)

\( \theta = 45^\circ \)

<table>
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<th>( t )</th>
<th>( \cos(\omega t) )</th>
<th>( \cos(\omega t + \frac{\pi}{4}) )</th>
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</table>
\[ P_{\text{tot}} \text{ (julies/sec). at some radius } r, \text{ we have that the} \]
\[ \text{total power per area, } S, \text{ is} \]
\[ S = \frac{P_{\text{tot}}}{4\pi r^2} \]  
\[ \text{area of spherical surface} \]

an absorbing film of area \( A \), thickness \( t \), and density \( \rho \) sits at radius \( r \) from the sun. It feels a gravitational force inward due to its mass

\[ F_g = \frac{G M (A + \rho)}{r^2} \quad (A + \rho \text{ is volume}) \]

it feels a force from the radiation pressure pushing it outward.

\[ P_r = \text{Pressure} = \frac{1}{c} S \]

\[ F_r = P_r A = \frac{1}{c} S A = \frac{P_{\text{tot}} A}{4\pi r^2} \]

the radiation pressure balances the gravitational force, and the sail is "suspended" against falling when

\[ \frac{G M (A + \rho)}{r^2} = \frac{P_{\text{tot}} A}{4\pi r^2} \]

the area of the sail, and the distance from the sun drop out. at any distance from the sun, the sail will be accelerated by radiation pressure if

\[ t < \frac{P_{\text{tot}}}{4\pi c GM \rho} \]

for aluminized mylar:

\[ \rho = \frac{2 \text{gm}}{\text{cm}^3} = 2 \times 10^3 \text{kg/m}^3 \]

\[ t < \frac{4 \times 10^{26}}{(4 \pi \times (3 \times 10^8)(6.67 \times 10^{-11})(1.99 \times 10^{30} \cdot 2 \times 10^3)} \]

< 0.4 \mu \text{m, pretty thin!} \]
This analysis also sets the size for small particles which can hover around the sun. We can approximate spherical particles as cubes, and the critical thickness above is their diameter.

For ice, \( \rho = 19 \text{ g/cm}^3 \),

\[
\tan \theta \approx 0.8 \mu
\]

Particles of ice with diameter \( \approx \mu \) "hover" at any radius from the sun.

\[
R = \frac{\langle S_r \rangle}{\langle S_i \rangle} = \frac{\frac{1}{2} \rho \varepsilon_0 E_r^2}{\frac{1}{2} \rho \varepsilon_0 E_i^2} = \frac{\frac{1}{2} \rho \varepsilon_0}{\frac{1}{2} \rho \varepsilon_0} \left[ \frac{\sqrt{\varepsilon_r - \sqrt{\varepsilon_r \varepsilon_0}}}{\sqrt{\varepsilon_r + \sqrt{\varepsilon_r \varepsilon_0}}} \right]^2 = \left( \frac{\varepsilon_r - \sqrt{\varepsilon_r \varepsilon_0}}{\sqrt{\varepsilon_r + \sqrt{\varepsilon_r \varepsilon_0}}} \right)^2
\]

In the brackets, multiply top and bottom by \( \sqrt{\varepsilon_r} \).

\[
R = \left[ \frac{1 - \sqrt{\varepsilon_r \varepsilon_0}}{1 + \sqrt{\varepsilon_r \varepsilon_0}} \right]^2
\]

Now recall \( \frac{\varepsilon_r}{\varepsilon_0} = N \Rightarrow N = \text{index of refraction} = \sqrt{\frac{\varepsilon_r}{\varepsilon_0}}\). For \( N > 1 \), \( N = \sqrt{\frac{\varepsilon_r}{\varepsilon_0}} \). Thus

\[
R = \left[ \frac{1 - n}{1 + n} \right]^2
\]