Assignment 2: Oscillators, Damped Oscillators
Due 9/24 in class, or in Amidei mailbox by 5pm.

Problems:

1. French 3-4. Do not copy the answer from the book. Derive the answer.

2. A rod of mass M and length L hangs from an eyelet screwed in one end, and can swing like a pendulum. Find the frequency of this physical pendulum for small oscillations.


4. Hydrogen chloride is an ionic molecule held together by the electrostatic attraction of H\(^+\) and Cl\(^-\). Assume the heavier Cl is stationary and the H ion orbits around it at a radius r. The electrostatic potential energy of the system includes the Coulomb attraction, and also, at small r, a strong Coulomb repulsion between the positively charged nuclei. The repulsive term is found empirically to vary as \(r^{-9}\). The total potential energy of the system can then be written as

\[
V(r) = -\frac{e^2}{4\pi\varepsilon_0 r} + \frac{A}{r^9}
\]

where A is to be determined.

a) Show that there is a radial separation, \(r = R\), where the potential is minimum, and find the value of R in terms of the other constants. If we consider this as the stable position of H, we may think of R as the inter-atomic spacing.

b) Show that for small radial oscillations of the H atom about this minimum, the motion will be simple harmonic, with frequency

\[
\omega_0 = \sqrt{\frac{2e^2}{\pi\varepsilon_0 R^3 m_H}}
\]

. c) The H and Cl ions form a dipole, and oscillations of the separation distance will cause a harmonic variation in the dipole moment. Later in the course we will see that a harmonically varying dipole moment emits electromagnetic radiation at the oscillation frequency. It is found that HCl indeed emits infrared light at approximately \(10^{14}\) Hz. Use this fact to find the approximate inter-atomic spacing, R, in HCl.

d) Extra credit: Find A. If you have plotting tools, graph the function. Verify the existence of a minimum at your value of R.
5. A capacitor C, inductor L, resistor R, and a switch are connected in series. The capacitor is charged to a value \( q_0 \), and at \( t=0 \), the switch is closed.
   a) Write down the differential equation for this circuit. Use complex notation to solve for the charge on the capacitor as a function of time, \( q(t) \).
   b) Assume weak damping. At what frequency does the circuit oscillate? At what time is the charge \( q \) reduced by \( 1/e \) of its initial value?
   c) Find the energy in the inductor and capacitor as a function of time. For the inductor, assume very weak damping and simplify. Show that the total energy decays exponentially in time, and find the time constant. Compare to part b. Make a careful sketch of both energies and total energy vs. time on the same plot.
   d) Calculate the Q of this oscillator.

6. A mass \( m=0.2 \text{kg} \) is hung from a spring with spring constant \( k = 80 \text{ N/m} \). The mass can be dipped in a viscous fluid that supplies a dissipative drag force given by \( -bv \), where \( v \) is the velocity (m/s).
   a) Set up the equation of motion
   b) The oscillation frequency with damping is \( \sqrt{3}/2 \) of the frequency with no damping. What is the Q of the system?
   c) The damping is increased until the system is critically damped, \( 2\sqrt{k/m} = b/m \). Show by direct substitution in your differential equation that the solution is
      \[
      x(t) = (A + Bt)e^{-bt/2m}
      \]
   d) If the critically damped oscillating mass starts at \( t=0 \) from the origin with \( v=3 \text{m/s} \), what is the maximum displacement and when does it occur? Plot the displacement and velocity with time.

   You may find some of the applets on the course Web page useful for this problem

7. French 4-6

8. French 4-12